COOPERATION TYPES SPECIFIC FOR BARRIERS

Diemut Lange

Leibniz Universität Hannover

We present the results of a study analysing the cooperative behaviour of fifth-grade student pairs when they have to overcome a difficulty (a barrier) in a mathematical problem task. As analysing method we used content analysis as well as frequency-tests (χ^2-test, CFA, Freeman-Halton-test). Our results help to confirm and elaborate the vague assumptions about cooperation types occuring at barriers based on the results of existing studies. In addition, our results suggest that 'presenting an idea of how or why to do sth.' could be relevant for overcoming the barrier in a pair.

It could be useful to solve a mathematical problem in a group: "The reasons given for the use of group work in problem solving include the opportunity for pooling of ideas, the natural need that arises to explain and express ideas clearly, and the reduction in anxiety for tackling something hard" (Stacey 1992, p. 261). If students reach a difficult point, they can stimulate and encourage each other to elaborate and question ideas. But surprisingly Stacey's study (Stacey 1992) as well as other studies suggest that there doesn't exist such an easy connection like 'groups solve math problems better than individuals'. This study will provide a more nuanced view to this phenomenon by describing the cooperation types specific to problem solving.

THEORETICAL FRAMEWORK

In mathematics as well as in psychology, a *problem* is understood as a task in which the problem solver has to overcome at least one difficulty or barrier (e.g. Dörner 1979). In contrast to a problem, in our study a *routine task* is taken to be a task without such a barrier for the problem solver. Thus, if the task is a problem or a routine task *depends on the problem solver* who tries to solve the task (e.g. Schoenfeld 1985). The term barrier is circumscribed vaguely in the literature (e.g. Schoenfeld p. 74: "intellectual impasse"). In our study a *barrier* is defined as a passage in a solving process where a solver does not perform something self-evidently and cannot remember essentials for solving. For example a person hesitates what to do next or questions the last ideas. For solving the problem the solver can work heuristically.

In our study we are interested in describing the kind of cooperation at barriers (for our understanding of cooperation see below). Only a few studies differentiate between separate cooperation types (e.g. asking, checking, explaining) and connect them with a difficulty in a problem. The research results suggest that there might be cooperation types specific for barriers and hints at these cooperation types.

Gooding and Stacey (1993) analysed cooperation-processes when students were working on difficult tasks in mathematics. For that, they modified a coding system which Sharan and Shachar used for tasks in geography and history. Gooding and

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Stacey coded the categorie *thinking aloud* more frequently than Sharan and Shachar. The researchers explained this difference with the distinctions in the nature and the difficulty of the tasks: In order to solve these tasks students have to spend more time in understanding difficult ideas and in stating a problem over and over.

Apart from this cooperation type, students possibly cooperate at barriers more frequently in types which focus on the 'how' (e.g. explanations, elaborations and demonstrations) rather than in types which focus on the 'what' (informative) or on the 'why' (evaluative). So, Hertz-Lazarowitz (1989) compared the cooperation types when students were working on a low-cooperative task with the types when students were working on a high-cooperative task (working on the task-process together). If the task was interpreted as high-cooperative, the students discussed most frequently on the how-level.

In addition, there might be a difference in cooperation types if only one student has a barrier or if both students have the barrier. So, Goos et al. (1996) observed that if the relative expertise of the students is unequal, the dominant cooperation type is *peer tutoring* – in the other case *collaboration* (if there is a degree of challenge for the students inherent in the task).

In sum, we hypothesize that there might be cooperation types specific for barriers. So the research results give hints for these cooperation types (*thinking aloud*, *peer tutoring* or *collaboration*). We also assume that students who have to overcome a barrier interpret the task as high-cooperative. So students are expected to cooperate at barriers most frequently on the *how-level*. In addition, the distinction 'barrier only for one student' and 'barrier for both students' might be suitable for our research purpose. But the assumptions based on these research results remain superficial, since the researchers use different definitions of the terms barrier and cooperation.

Consequently, the research questions explores the characteristics of cooperation types specific for barriers. In order to get precise results, we will differentiate between cooperation types that (a) occur only at barriers and not in the rest-process, that (b) can be found more frequently at barriers than in the rest-process, and that (c) appear both at barriers and in the rest-process but bear special characteristics when employed at a barrier.

As a system for cooperation types we don't use one of the systems in these three studies because the differentiated types are either too rough for describing the cooperation behaviour at barriers (Hertz-Lazarowitz and Goos et al. differentiate only between three different cooperation types) or not suitable to model the whole cooperative behaviour in the own data (Gooding & Stacey). So, we build upon Naujok's definition of cooperation (Naujok, 2000, p.12: "every kind of task-related interaction") and her task-related cooperation types *explaining, asking, comparing, prompting* and *copying*: Her understanding of *cooperation* as an "empirical phenomenon" (p. 12) is more useful for our study than the definition as an ideal way to work together (Slavin, 1983), since first of all we aim at describing the cooperation

behaviour without a normative direction and since we expect various kinds of cooperation when solving mathematical problems. In a former study (Lange, 2012) we modeled the variety of cooperation acts when solving mathematical problems. We adapted and completed the list of cooperation types (see below).

DESIGN AND METHOD

Study: Between November 2008 and June 2010 we organized a math club at the University of Hanover (MALU), an enrichment project for fifth-grade students (age 10-12). In this math club the students had to solve one or two mathematical problem tasks in pairs one afternoon a week. After working in pairs, we discussed possible ways to solve these tasks with the whole math club group. Based on the results of two tests (a general giftedness test CFT-20R, a mathematical giftedness test) we selected a group of 9 to 14 fifth-graders with different results for each of the four MALU semesters (for more details concerning the tests see Gawlick & Lange 2011). Since the kind of cooperation may depend on the composition of a pair, we arranged homogeneously as well as heterogeneously composed pairs (criterium: test results), pairs which were held constant during the semester as well as pairs with changing partners.

The problem solving processes were videotaped and the students' notes were collected. In addition, a log was kept of the children's main thoughts and the observers' subjective impressions.

Tasks: Cooperation and the kind of barrier possible for fifth-graders in the tasks may vary with different problem tasks, so we looked at task-collections (e.g. competition tasks) and problem solving books, analysed the tasks and chose tasks with different features. Fifth-graders should be able to solve the chosen problem solving tasks with their mathematical knowledge. Two of the tasks we presented are the following:

The seven gates (Bruder 2003, p. 12)
A man picks up apples. On his way into town he has to go through seven gates. There is a guardian at each gate who claims half of his apples and one apple extra. In the end the man has just one apple left. How many apples did he have first?

Oh yes, the chessboard (idea: Mason et al. 2010)
Peter loves playing chess. He likes playing chess so much, that he keeps thinking about it even when he isn't playing. Recently he asked himself how many squares there are on a chessboard. Try to answer Peter's question!

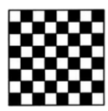

Figure 1: Two problem tasks

Evaluation Method: Because of the large amount of data and in order to receive a maximum diverse sample of cooperation types and barriers, we selected a cross-section from the data (different pairs, different semesters, different tasks) – altogether 23 processes (the analysing method is very time-consuming). The videographies of the processes were transcribed, and the transcripts were revised before the coding process started. As coding method for identifying cooperation acts and barriers we used qualitative content analysis (Mayring 2008).

In reference to cooperation, we were able to build upon Naujok's cooperation acts and upon her descriptions of the cooperation phenomenon (see above). Since the setting in Naujok's investigation (routine tasks, tasks from different subjects in school, younger students, school setting) differs in some kinds from our setting, the cooperation acts were collected on the one hand deductively from her study and on the other hand empirically from the MALU-transcripts (for more details in the development of the coding system see Lange 2012). The following table represents an excerpt from the original coding system for cooperation types (some cooperation types are explained together). An important point is the differentiation between the cooperation on the *what-, how-* and *why-level*. The *why-level*-cooperation in our study involves giving reasons and evaluating something. If students discuss products like results / answers or part of results / answers, they cooperate on the *what*-level – if they are interested in possibilities to understand or solve the task, they cooperate on the *how-level*. In our transcripts students use the terms *how* and *why* not selectively, partly even synonymously, that means they ask for example for reasons (*why*), but get an answer on the how-level ("first I did ..., then I did ..."). Therefore we cannot differentiate between these two levels (how- and why-level). If the interaction takes place visually, we defined this as *non-verbal*, otherwise as *verbal*.

presenting an idea	One person mentions an idea about a possible solution or about possible parts of the solution path (e.g. next steps, task-features, argumentation) before this person has solved the task or the parts of the solution path, which the person presents.
informing about sth.	One person says her/his (part) solution or says how she/he solved the task or parts of the task.
helping	One person has an advance in knowledge belonging to a part of the task and shares this knowledge with a partner within this cooperation type. The person can write it down (*passing sth. non-verbally*) or *prompt* or *explain* it. The partner can also be active by copying information from the person's notations.
comparing sth.	At the moment of comparing the comparing persons have solved the part of the task, so they (or only one person) inform themselves about this part of the task ((part-)result, (part-)answer, process-step, whole process) of the other person.
evaluating, checking, pointing out a mistake	If persons check something, they go through the solution path and reflect the steps. Instead one person can *point out a mistake* or can assess the correctness or usefulness of something (*evaluating*). These three cooperation acts occur in the MALU-processes as *saying-what* as well as as saying-how. In the case of these acts the person could either have the solving step already done or haven't yet.
commenting on the task	The persons say something about the task (e.g. familiar, funny) or comment on the difficulty of the task (e.g. easy, difficult).
asking	One person asks sth., the partner responds. In order to reduce an overlap, this category is only coded, if the passage cannot be subsumed under another category.

Table 1: Excerpt from the coding system for cooperation types

The transcripts were coded in two steps: First, we marked the points if something changes (cooperation theme, cooperation type, cooperation into non-cooperation or reverse). Second, we dedicated a cooperation type-label to the marked out transcript-passages.

Belonging to barriers in the process, we started with the formulated definitions (see above) and coded first all barriers in the process. In a second step we decided, if only one person face the barrier or if both students experience the barrier. As signs for a barrier we observed among others the following aspects: A person says that she doesn't know or that she's unable to do something / A person does something not self-evidently (e.g. she/he hesitates or questions something). / A person changes the perspective or has an illumination.

We trained three students in coding with both category systems. For the first (marking the begins of new cooperation-phases) and the third decision (decision for a barrier) the pairwise interrater-agreement varies between 60% and 69%. For the second decision (labeling the marked cooperation-phases) Cohen's Kappa varies between 0.64 and 0.68. Also the percentual-agreement-values can be termed as good because we only considered those transcript-passages where at least one person coded a cooperation-change or a barrier. After coding independently, we discussed the decisions where we disagreed, and coded jointly.

For answering the second part of our research question (b) (if certain cooperation types occur more frequently at barriers than in the rest-process), we used the kx2-χ^2-test with the CFA (Configural Frequency Analysis) as a posthoc-test. Since the codings are independent of each other and since every passage can be coded clearly, two of the three test-assumptions are fulfilled. As the third test-assumption (at least 80% of the expected frequencies should be greater than or equal 5) is not fulfilled in our study (although it is disputed), we also did the Freeman-Halton-test as an exact test.

RESULTS

Among the 23 chosen processes for analysis we coded 38 barriers in 16 problem solving processes. At about 82% of the barriers, the students cooperated. In figure 2 the cooperation types occuring at barriers are marked in bold. The non-marked cooperation types occurred only in the rest-process. The superficial assumption that none of Naujok's cooperation acts (blue-grey background) but only the added cooperation types (white) could be observed at barriers proved incorrect. Comparing-types as well as some of the evaluating-types first of all help to uncover a mistake, so that these types do not occur in the context of discussing a barrier. The helping-types *copying, prompting* and *passing sth. non-verbally* are used to get information – often without questioning this information. However, the question remains whether the cooperation *type informing about how or why have done sth*. occur at barriers. Perhaps students only seldomly question these contents (see Goos 2002 for reasons for metacognitive failure).

Regarding the three above formulated criteria, we found the following results: None of the cooperation types were observed only at barriers (a). For analysing how far the second criterion (b) is fulfilled, we first summarized the cooperation types each belonging to *helping, presenting, informing, comparing, evaluating* and other types. This approach seems to be useful for testing purposes (otherwise: many cells with an

expected frequencies smaller than 5) and also possible because of the contentual relationship of the types belonging to the same cooperation intention. For this analysis we take all transcripts from two tasks (see fig. 1) into account, in order not to distort the quantitative results through sporadic chosen problem solving processes. See table 2 for results (expected frequencies in brackets).

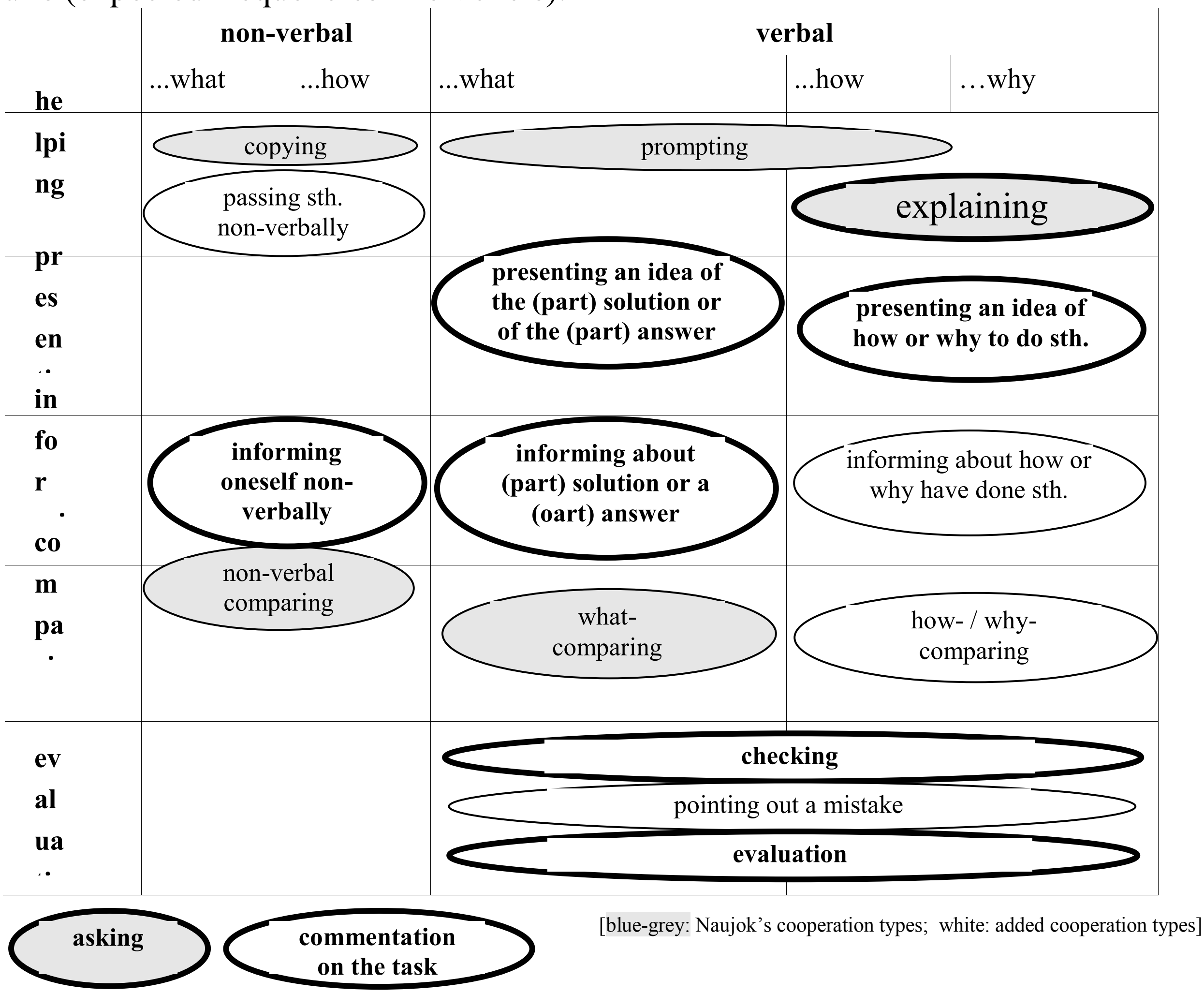

Figure 2: Cooperation types occuring at barriers (marked boldly)

cooperation intention	**barrier**	**rest-process**	**Σ**
helping	2 (2.81)	20 (19.19)	22
presenting	**14 (6.77)**	39 (46.23)	53
informing	0 (3.45)	27 (23.55)	27
comparing	0 (2.68)	21 (18.32)	21
evaluating	4 (4.60)	32 (31.40)	36
other types	3 (2.68)	18 (18.32)	21
Σ	23	157	180

Table 2: Contingency table – cooperation types at barriers vs. in the rest-process

The 6x2-χ^2-test as well as the Freeman-Halton-test gave a significant result: both-sided test, $\chi^2=16.28>11.1=\chi^2_{\alpha,df}$ $df=5$ (the significance level $\alpha=0.05$ is appropriate, because

the study is an explorative one). The effect is medium-sized ($w \approx 0.3$). The configuration *presenting / barrier* can be accepted as statistical ascertained (CFA-results: $\gamma \approx 0.0085 \leq 0.05 = \alpha = \alpha^*$). That means the cooperation intention *presenting* occurs significantly more frequently at barriers (14) than expected (6.77) (shaded in grey in tab. 2). By calculating the χ^2-values for both presenting-types separately, we found that the above mentioned statement is only true for the cooperation type *presenting an idea of how or why to do sth*. ($\chi_{ij}^2 == 14.46 > 3.84 = \chi_{0.05,1}^2$). So, only this cooperation type occurs significantly more frequently at barriers than expected.

With regard to the cooperation type *presenting an idea of how or why to do sth*. we had enough transcript passages in order to compare the appearance at barriers with the appearance in the rest-process (c). In the rest-process this type was coded when students proposed a procedure for solving the next steps or when students considered or implemented an idea. On the contrary at barriers this type was coded when students didn't know how to proceed next or how far the previous ideas were appropriate for solving the problem. In sum, based on this cooperation type we found manifestations occuring only at barriers and others we found only in the rest-process.

DISCUSSION AND CONCLUSIONS

Our results are compatible with and elaborate the results found in the three studies referenced in the theoretical section. Gooding and Stacey's cooperation type *thinking aloud* contains facets from both presenting-cooperation acts. Both types occur at barriers (fig. 2), but as elaboration of their result only one type, namely *presenting an idea of how or why to do sth.*, appeared more frequently at barriers than expected.

The assumption that barriers occur more frequently in high-cooperative than in low-cooperative tasks is correct for our data: For that we recoded the MALU-transcripts in terms of low- and high-cooperative-processes. In accordance with Hertz-Lazarowitz' results, at barriers students cooperated most frequently on the how-level. Our results offer two specifications: Only the type *presenting an idea of how or why to do sth.* represents the type the students cooperated in most frequently. In addition, students cooperated at barriers not only on this level, but rather on the other two levels.

Also with regard to the results of the third study, our results can help to differentiate the statements: So, we observed not only helping-types if only one person has the barrier, but also for example presenting-types. Further research is needed concerning the differentiation between a barrier only for one and a barrier for both students: Our results couldn't confirm the results found by Goos et al. (1996). One reason might be a (possibly too) small data-base for this differentiation in our study.

All together, our results confirm the vague studies' results. Beyond this, our system allows for more specific predictions because our types are finer with more facets. In addition, we now can make a statement about the types at barriers as well as about quantitative connections between cooperation acts and the occurrence of barriers.

Our results suggest two additional conclusions: Firstly, because of our results to the first criterion (a), one cannot decide if students contemporarily have to overcome a barrier only based on the cooperation types. But the results to the third criterion (c) let us assume that this is possible when we distinguish between different manifestations related to each cooperation type. Secondly, our results emphasize the relevance of the cooperation type *presenting an idea of how or why to do sth*. for problem solving in pairs, because amongst all cooperation types at barriers this cooperation act appeared at barriers most frequently and furthermore significantly more frequently at barriers than in the rest-process. With a view to learning theories, cooperation in this type might help to overcome the barrier in pairs.

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